Why quantum computing is hard
– and quantum cryptography is not provably secure

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Abstract

Despite high hopes for quantum computation in the 1990s, progress in the past decade has been slow; we still cannot perform computation with more than about three qubits and are no closer to solving problems of real interest than a decade ago. Separately, recent experiments in fluid mechanics have demonstrated the emergence of a full range of quantum phenomena from completely classical motion. We present two specific hypotheses. First, Kuramoto theory may give a basis for geometrical thinking about entanglement. Second, we consider a recent soliton model of the electron, in which the quantum-mechanical wave function is a phase modulation of a carrier wave. Both models are consistent with one another and with observation. Both models suggest how entanglement and decoherence may be related to device geometry. Both models predict that it will be difficult to maintain phase coherence of more than three qubits in the plane, or four qubits in a three-dimensional structure. The soliton model also shows that the experimental work which appeared to demonstrate a violation of Bell's inequalities might not actually do so; regardless of whether it is a correct description of the world, it exposes a flaw in the logic of the Bell tests. Thus the case for the security of EPR-based quantum cryptography has just not been made. We propose experiments in quantum computation to test this. Finally, we examine two possible interpretations of such soliton models: one is consistent with the transactional interpretation of quantum mechanics, while the other is an entirely classical model in which we do not have to abandon the idea of a single world where action is local and causal.

1 Introduction

Quantum computation appears straightforward at small scales of two or three qubits, but attempts to scale it up have not been successful. Shor showed in 1994 that large-scale quantum computers could have significant impact, such as in factoring the large integers that form the basis of RSA cryptography [1], but this would require maintaining coherence among thousands of qubits. In 1998, Jones, Mosca and Hansen reported a quantum computer with two qubits [3]

while Chuang, Gershenfeld, Kubinec and Leung demonstrated a cascade of three [4]. In 2001, Vanderspysen, Steffen, Breyta, Yannoni, Sherwood and Chuang reported a quantum computer that could factor 15 [5]. In 2002, the Los Alamos quantum information science and technology roadmap aimed at having functioning quantum computation testbeds by 2012 [6]. See Chen et al [7] for an extensive survey of the technology. Yet despite the investment of tremendous funding resources worldwide, we don't have working testbeds; we're still stuck at factoring 15

using a three-qubit algorithm [8].

It is time to wonder whether there might be something we missed, such as theoretical limits on entanglement and coherence. Doubts about the feasibility of quantum computers go back to 1995, when Unruh warned that maintaining coherence might be hard [2]; researchers in this field still see the problem in terms of reducing sources of noise (for example by using lower temperatures), on increasing the signal (for example by bringing the particles closer together) and on using error-correcting codes [7, 9]. Researchers are now starting to wonder whether geometry affects entanglement and coherence; the first workshop on this topic was held last year [10]. However, experiments elsewhere in physics suggest a type of limit that has not so far been considered.

2   Guiding waves

In recent experiments by Couder and colleagues [11, 12, 13, 14, 15], a small liquid drop is kept bouncing on the surface of a bath of the same liquid by oscillating this substrate vertically. The bouncing induces waves in the surface which, in certain regimes, guide the motion of the droplet. As shown schematically in Figure 1, in this regime the droplet moves along the surface at the same velocity as the peaks and troughs of the waves in the vicinity.

By measuring the statistical motion of the droplet, the experiments show clear phenomena corresponding to those of quantum mechanics, including single-slit diffraction, double-slit diffraction, quantised energy levels and tunnelling through a barrier. A video shows clearly how quantum-mechanical phenomena can arise in a completely classical system [16].

Figure 1: Schematic of droplet phase locked with surface waves

In this two-dimensional analogue there is a limit to the number of qubits in a coherent system. It is easy to get phase coherence with waves associated with one other particle and possible to get coherence with two – one coherence per dimension. (In a three-dimensional system, a further coherence could be added.) Kuramoto and others have developed extensive mathematical models of coupled oscillators; for a review, see Acebrón et al. [17]. It is the Dangelmayr-Knobloch radial standing-wave solutions that appear of most interest here [18]. Even so, a single coherence between an ensemble of particles is more likely, so that they will act as a single ensemble, as when the many electrons in a Josephson junction act as a single qubit. (Coupled-oscillator models have already helped explain other aspects of Josephson junction behaviour.)

Couder's experimental measurements are also evocative of the de Broglie–Bohm model of quantum mechanics [19, 20, 21], which is equivalent to the traditional Copenhagen interpretation. In this model, a small particle interacts with waves in three dimensions which obey the same equations as the quantum mechanical wavefunction. The motion of the particle is given by

$$v \; = \; \frac{\hbar}{m} \, Im\left(\frac{\nabla\psi}{\psi}\right) \qquad (1)$$

and the resulting observables are the same as those of the Copenhagen interpretation; in fact equation (1) is merely the equation that is required for this to happen (it is derived from the usual quantum mechanical wavefunction plus a continuity condition). The models are also equivalent for a quantum mechanical system with entangled states. Indeed Nikolić has argued that had the Bohm interpretation come along first, no-one would have needed the Copenhagen interpretation [22]. But the de Broglie–Bohm model may give more insight into what happens when a system loses coherence.

If two particles are entangled, then the guiding wave ψ of one particle must be correlated with that of the other. Now as quantum wavefunctions are considered to be nonlocal, this caused difficulty for some writers: Bell, for example, ar-

gued that the nonlocal nature of the wavefunction of two spin-1/2 entangled particles meant that a geometrical interpretation of the guiding wave was impossible [21]. The textbook approach is that in such circumstances the guiding wave is in six-dimensional configuration space, for which a geometric interpretation in physical space is not obvious. Yet Bell also warned that impossibility proofs mostly represented a failure of imagination, and he himself had demolished previous arguments against a local-realist interpretation of quantum mechanics.

We will argue, first, that the loss of phase coherence may provide a better model for the behaviour observed in quantum decoherence experiments; and second, that this hypothesis might be tested by decoherence experiments that measure the physical geometry associated with entanglement and decoherence. Before that, we will discuss how soliton models might provide some insight into possible underlying mechanisms, in order to tackle the imagination failure. By presenting a local-realist model that is consistent with de Broglie–Bohm and with observed empirical results, we challenge the argument of impossibility.

3   Soliton models

Solitons are persistent, localised solutions of the wave equation (with additional nonlinear terms, which are usually small). They arise in fluid and other media, having first been observed and described on a canal in the mid-19th century [23], and were applied to particle physics following the proposal by Skyrme in 1961 of a model of an atomic nucleus, later developed and popularised by Witten [24, 25]. Many other soliton models have been proposed in various branches of physics. More recently, for example, Volovik has found that quasiparticles in liquid helium exhibit many of the properties described by the Copenhagen model and relativity (albeit with c being the speed of sound in the fluid) [26], and raised the question of whether fluid models could be applied to all elementary particles.

In the field of analogue gravity, Unruh and others have explored fluid models of black holes [27] and this led to a thriving research programme exploring many provocative analogies between fluid flow and general relativity [28]. In particular, an event horizon corresponds to the start of supersonic flow; Lahav and colleagues have observed this experimentally in a Bose-Einstein condensate [29]. In short, over the past thirty years, fluid models have developed to express most of the properties of elementary particles from the basic Copenhagen model to (in aggregate) general relativity.

In a companion paper, Brady has proposed a soliton model for the electron [30] which we will now summarise. It provides a fluid-model analogue of the Coulomb force, and is thus of relevance at least to decoherence in quantum computers relying on electron behaviour (such as qubits based on Josephson junctions). The key insight is that Euler's equation for a compressible fluid possesses quasiparticle solutions with chirality. These may be visualised as smoke rings but with a twist, in that the line of greatest pressure circulates not merely around the ring's long diameter but around its short one too.

Consider a compressible inviscid fluid of pressure P, density ρ and velocity $\mathbf{u}$ of an inviscid fluid medium that obeys Euler's equation:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\frac{1}{\rho}\nabla P \qquad (2)$$

where $\partial \rho/\partial t = -\nabla(\rho\mathbf{u})$. At low amplitude, this gives the wave equation

$$\frac{\partial^2 \rho}{\partial t^2} = c^2 \nabla^2 \rho \qquad (3)$$

The wave equation has linear solutions, and also eddy-like solutions like smoke rings. There the line of greatest density rotates round the ring's small axis, as in Figure 2a. However, there are also chiral solutions where the line of greatest density rotates around both axes, as in figure 2b. The general solutions are referred to as sonons. This solution of the wave equation can be written

$$\xi_{mn} = \psi_o R_{mn} \qquad (4)$$

where

$$\psi_o = A e^{-i\omega_0 t} \tag{5}$$

$$R_{mn} = \int_0^{2\pi} e^{-i(m\theta' - n\phi)} j_m(k_r\sigma) k_r R_o d\phi \tag{6}$$
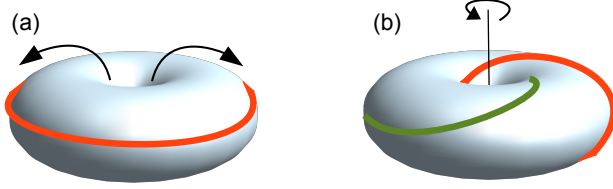

Figure 2: Sonons (a) without chirality (b) with chirality

Figure 2a shows the R_{10} sonon. The red line is the line of maximum density, rotating at angular speed ω_0. Figure 2b shows the R_{11} sonon, which models the electron. In such particles, the chirality, spin direction, m and n are preserved by continuous transformations, so are persistent and quantised. At low amplitude they are Lorentz covariant because they obey the wave equation (3), which is Lorentz itself covariant, and it turns out that the perturbations at finite amplitude average to zero over a cycle. Classical dynamics follow in the approximation of constant R_{mn} and small v/c. Meanwhile, at a large distance from the sonon, χ may be approximated up to a phase factor as

$$\chi = \frac{1}{r} \sin k_r r \tag{7}$$

(We refer the reader to [30] for the details.)

The important point for this paper is that χ behaves like a carrier wave and ψ as its modulation, which is a complex function as its phase is important. This provides a physical model of the de Broglie–Bohm view that a particle moves through space surrounded by waves that obey the usual quantum equations. Extending equation (5) into a Lorentz covariant form leads directly to the Klein–Gordon equation

$$\frac{\partial^2 \psi}{\partial t^2} - c^2 \nabla^2 \psi = -\omega_0^2 \psi \tag{8}$$

(the relativistic form of Schrödinger's equation); with a little more work we find that the R_{11} sonon is governed by the Dirac equation, which describes the behaviour of the electron in detail [30]. It follows that provided a system remains coherent, the usual predictions of quantum mechanics will apply. (The analogue gravity community has found numerous cases of quantized behaviour of sound waves in fluids and applied them as analogies to other problems in quantum physics; see the survey by Barceló, Liberati and Visser [28].)

The more detailed equations (4–7) enable us to make a number of predictions about decoherence. For example, as the carrier wave χ decays as $1/r$, the system will be more prone to decoherence with distance.

In the absence of decoherence, the equations of motion are time-reversal symmetric, as Euler's equation is. The state of the system at any one time determines its state at any other time, whether in the future or in the past. Thus it might not be surprising if we see behaviour that appears to violate microcausality [31]; entropy kicks in once phase coherence is lost. The big question is whether we can have a local realist model of quantum systems without violation of macrocausality. This leads us to Bell's theorem.

4 Local realism and quantum crypography

If the soliton model of the electron (or perhaps another coupled-oscillator theory) is correct, then two of the possibilities are as follows.

Weak (transactional) soliton hypothesis:
the elementary particles are solitons in an inviscid fluid, but time reversal symmetry in entangled states means that there may be violations of microcausality. We still get quantum electrodynamics with advanced and retarded waves following the exposition of Mead [32], and relativity works because all particles are solutions to the wave equation and thus Lorentz covariant.

Strong (causal) soliton hypothesis: the elementary particles are solitons in an inviscid fluid; relativity emerges from the fact they satisfy the wave equation; and quantum mechanics from the nature of the solutions. So Euler's equation explains not just the motion of matter, but also electricity, light and atomic forces.

These two interpretations give quite different views of reality. The first is analogous to Cramer's transactional interpretation of quantum mechanics [33]. The second is a classical view of the world; Newton's laws determine everything, including the very large and the very small.

Initially one might think that Bell's theorem, and the entanglement experiments inspired by it, compel us to favour the former. But a closer examination suggests that this is not necessarily so, because the experiments are designed to interact with the propagating waves, not, on this hypothesis, with the carrier waves which might themselves carry information about spin correlations.

If an experimenter creates a pair of entangled particles, sends one of them round an optical fibre or waveguide or tunnel of length D, and then performs a measurement on them with equipment spaced a distance d apart for the two particles, then although the ψ waves of the soliton may have travelled a spacelike separation D, this does not necessarily hold for the χ waves whose phase coherence creates the entanglement in the soliton model. The χ waves are broadcast in all directions from a sonon and thus the distance that matters to prove impossibility results about coherence is d. If this is not spacelike then no violation of locality (or relativity or causality) has been proved.

In 1982, Aspect, Dalibard and Roger tried to exhibit a spacelike separation by using polarisers that switched in 10ns while the length L of the path traversed by the photons had $L/c =$ 40ns [34]. Yet they used a single receiver for coincidence monitoring, so $d = 0$.

In 1998, Tittel, Brendel, Zbinden and Gisin demonstrated coherence in photons sent round a 10.9km optical fibre in a direct attempt to probe the tension between quantum non locality and relativity; yet the same issue arises with this experiment [35]. The source, located in Geneva, was 4.5 km from the first analyser in Bellevue and 7.3 km from the second in Bernex, with connecting fibers of 8.1 and 9.3 km. However, entangled states were studied only when both photons went either through the short arms or through the long arms.

In the same year, Weihs, Jennewein, Simon, Weinfurter and Zeilinger performed an experiment with what they believed was a proper spacelike separation: photon pairs were sent from a source to two detectors 400m apart and were found to be coherent on arrival [36]. However this does not establish that information was transmitted faster than light by the ψ wavefunction, as coherence is maintained by the χ wave which travels at the speed of light just like the photons but in a straight line.

In 2008 Salart, Baas, van Houwelingen, Gisin and Zbinden did a fibre-loop experiment over a distance of 18km (from Geneva to Satigny and Jussy) and actuated a piezoelectric crystal which moved a mirror, ensuring that coherence was lost [37]; yet the same applies here as in Weihs' experiment.

In short, experimenters have sought to close one loophole after another in the Bell test experiments over the last thirty years. But the soliton model of the electron creates another major hole as the experimenter must consider not just the propagation of the quantum-mechanical wavefuntion ψ but also of the density waves χ on which they are modulated.

The consequences for quantum crypto are notable. As the experiments done to test the Bell inequalities have failed to rule out a classical hidden-variable theory of quantum mechanics such as the soliton model, the security case for quantum cryptography based on EPR pairs has not been made.

We propose that experimeters test explicitly whether entanglement is a function of physical

geometry in the way predicted by the soliton model, or more generally by the results of Kuramoto theory.

First, one might fabricate a series of 3-qubit quantum computers with the coherent elements in a triangle whose largest angle was 90°, 100°, ..., 180°. We predict that 3 distinct qubits will not be measured when the elements are collinear, and perhaps also when they are nearly collinear. One might also make a 4-qubit machine in three dimensions, and similarly measure the correlation with geometry.

Second, more general entanglement experiments might attempt to identify behaviour consistent with Kuramoto theory such as finite size effects on decoherence, relationships with the order parameter and whether bifurcation points can explain the circumstances in which systems become coherent.

Third, we suggest close scrutiny of claims that computation can be sustained without decoherence. If the strong soliton hypothesis is correct, we would expect that a single physical qubit cannot be recycled in the same coherent computation; thus if a computation requires k steps on n qubits it would need at least a k-by-n array of qubits, not a single k-qubit register plus some CNOT gates. If quantum mechanics is really just a convenient calculus for dealing with coupled oscillators, then reality is classical, and quantum computers are just classical computers. They cannot then provide a way to beat the Bremermann limit of mc^2/h computations per second for a computer of mass m [38].

5 Conclusions

One of the big puzzles that straddles the boundary between physics and computation is why quantum computers have got stuck at three qubits. We have shown that a local-realist version of the de Broglie–Bohm interpretation of quantum mechanics provides a good explanation: entangled particles are precisely those whose guiding waves are phase coherent. It follows that we can expect two entangled qubits

to be possible on a line, three in a plane and four in a three-dimensional structure. In fact, it may be more helpful to model qubits as coupled oscillators, following Mead's model of quantum electrodynamics and Kuramoto theory, than using Hilbert space. We propose experiments to verify this directly.

Bell warned that claimed impossibility proofs often showed merely a lack of imagination on the part of the 'prover', so we presented a concrete guiding-wave model given by a recent soliton model of the electron. In this model, the electron is a spinning twisted torus in an inviscid fluid. It generates compression waves χ which are in turn modulated by guiding waves ψ.

Since the Bell test community has not yet considered the possibility that coherence information might be transmitted other than by the quantum mechanical wavefunction ψ, the experiments that have claimed to demonstrate non-local behaviour of entangled systems have done nothing of the kind. If entanglement is simply phase coherence, it is not enough to show that two photons sent to separated sensors remain coherent even though the distance between the sensors have a spacelike separation, as the phase coherence is carried by the χ waves. In consequence we dispute the claim that a quantum cryptosystem based on EPR pairs must be secure. The evidence needed to support that has simply never been exhibited.

We also challenge experimentalists who believe that entangled states violate locality to devise an experiment where locality fails in the soliton model. In fact since quantum mechanics and relativity can both be derived from this local and causal model, it will be surprising if anyone can use Bell's theorem to prove an incompatibility betweeen quantum mechanics, relativity, locality and causality, regardless of whether the soliton model turns out in the end to be the right one. More generally, we invite experimentalists to investigate the physical geometry of entanglement and coherence. The real prize is not the ability to build better quantum machines, but the far greater one of understanding the most fundamental questions. Do soliton models provide a

better explanation of the world than string theories? If so, which soliton models are supported? And in the absence of evidence, we need not accept that physics really requires us to abandon the concept of a single objective universe where action is both local and causal.

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